A QUANTUM BASIS FOR THE RELATIVISTIC DOPPLER EFFECT FOR LIGHT^{*)}

Daniel L. Alkon

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In the first of two derivations of the relativistic Doppler effect, obtained without directly applying the Lorentz transformations, the principle of relativity is used to introduce a probabilistic frequency-squared term and, thus, macroscopic uncertainty into the classical Doppler equations. In the second derivation, the classical Doppler effect for a moving mirror is broadened when introduction of Planck's law for radiation from a light source includes high radiation frequencies and thus high electron velocities $(V \rightarrow c)$. These methods are suggested to more directly describe intrinsic probabilistic properties of electromagnetic radiation rather than measurement differences attributable to a moving frame of reference.

Introduction. In the realm of atomic and subatomic particles, limits of measurement, independent of the measuring devices available, define essential uncertainties. Such uncertainties often necessitate physical descriptions that involve probability distributions for quantum states. In the realm of macroscopic phenomena, many of which are accessible to human perception, there is a "macroscopic" uncertainty, an uncertainty that can often be overcome by additional information. The principle of relativity, for example, includes an unavoidable ambiguity between a moving observer and a stationary frame of reference. In the absence of additional spatial and/or temporal information (such as those provided by a third, designated "stationary" frame of reference), an observer cannot discriminate his own movement at a constant velocity from the movement of a nearby frame of reference. It is, therefore, appropriate to distinguish "macroscopic" uncertainty from "microscopic" uncertainty often cannot be offset beyond certain limits of measurement regardless of the available measuring devices. Nevertheless, macroscopic uncertainty can include parameters that are statistical averages taken across probability distributions. Microscopic uncertainty, however, includes parameters obtained from quantum mechanical averages taken across probability distributions.

Einstein's special theory of relativity begins with macroscopic uncertainty and, via the Lorentz transformations, extends into the realm of atomic and subatomic particles and thus microscopic uncertainty. Extrapolation from the relativity of macroscopic movements yields accurate descriptions of a number of subatomic characteristics such as relativistic velocity, relativistic frequency, and relativistic mass as particle velocities approach the speed of light. However, since the same description may also be obtained, as decribed here, by applying principles of quantum theory, the latter may more accurately reflect the nature of the uncertainty that results when atomic and subatomic particles move with velocities that approach the speed of light.

According to the thesis examined here, when the special theory of relativity can be tested, it actually applies to microscopic uncertainty that falls within the domain of quantum theory. Based on the arguments that follow, the relativistic Doppler effect for light involves actual physical transformations of subatomic light

^{*)} The article is published as a polemical one. According to comments of Professor L. M. Tomil'chik, corresponding member of the National Academy of Sciences of Belarus, the article does not contain any new relations and does not predict any physical effects which are absent in the standard approach, and therefore the article can be considered only as an original technique.

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sources (e.g., electrons) as they approach the speed of light. Rather than frames-of-reference differences, the Doppler shifts of light frequency are suggested to more directly reflect velocity-induced energy shifts of light sources and their emissions within the quantum theoretic domain.

The theory of relativity of Einstein [1] broadens the classical principle of relativity to include electromagnetic waves by using the Lorentz transformations to take into account the inherent macroscopic uncertainty of moving frames of reference. Two different cases will be examined here in which the relativistic Doppler effect will be derived without directly applying the Lorentz transformations, but instead by considering:

1) inherent uncertainties (macroscopic) within moving reference frames;

2) inherent uncertainties (microscopic) of photon distribution and frequency that are intrinsic to electromagnetic-wave motion.

Case 1. Generalization of the Doppler effects for sound waves to classical Doppler effects for light waves must take into account that:

I) light requires no medium (i.e., ether) and, therefore, is considered to propagate in vacuo;

2) the actual speed of light (c) is constant and independent of movements of either the source (S) or the observer (O);

3) there should be no difference (measurable by O) between the case where S moves toward O or O moves toward S.

As a first approximation, the Doppler effect for sound waves and light waves (classical) can be derived from the fundamental relations between wavelength (λ), frequency (f), and relative velocity of motion (v) between S and O.

For a stationary observer O and a source S moving toward O, the source velocity reduces the waves at their origin, i.e., causes wave shortening, as specified by

$$\lambda = (c - v) T_{\rm p},$$

where v is relative velocity of S; T_p is the period; (c - v) is the wave speed relative to S. The new frequency f' is given by

$$f'_{S \to O} = \frac{c}{\lambda} = \frac{c}{(c - v) T_p} = \frac{c}{c - v} f$$

or

$$f'_{S \to O} = \frac{f}{1 - \frac{v}{c}} .$$

For a stationary S and an O moving toward S the moving observer encounters a greater number of waves (in unit time) and thus causes an increase in frequency given by

$$f_{\mathrm{O}\to\mathrm{S}}^{''} = \frac{c+v}{\lambda},$$

where (c + v) is the wave speed relative to the observer. Since $\lambda = \frac{c}{t}$, we obtain

$$f_{\mathbf{O}\to\mathbf{S}}^{''} = \frac{c+v}{c}f$$

or

$$f_{\mathcal{O}\to\mathcal{S}}^{''} = \left(1 + \frac{v}{c}\right) f.$$

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By similar arguments, when S moves away from O, we have

$$f_{\leftarrow \text{SO}} = \frac{f}{1 + \frac{v}{c}}$$

and when O moves away from S;

$$f_{\rm SO\rightarrow}^{''} = \left(1 - \frac{v}{c}\right) f.$$

Now for $v \ll c$ we obtain

$$f'_{S\to O} \approx f''_{O\to S}$$
.

However, as $v \to c, f' \neq f''$, as is readily apparent from comparisons of the binomial expansions for f' and f'':

$$f'_{S \to O} = \frac{f}{1 - \frac{v}{c}} = \left(1 + \frac{v}{c} + \frac{v^2}{c^2} + \dots\right) f , \qquad (1)$$

$$f_{\mathcal{O}\to\mathcal{S}}^{''} = \left(1 + \frac{v}{c}\right)f.$$
 (2)

The principle of relativity includes an unavoidable uncertainty encountered by the observer moving with a constant velocity. In the absence of other information the observer cannot discriminate his own motion from the motion of an observable stationary frame of reference. For our purposes here, this uncertainty of observer/frame of reference will be designated as "frame-of-reference" uncertainty. With respect to this unavoidable frame-of-reference uncertainty, Eqs. (1) and (2) adequately describe wave motion and frequency shifts for a moving light source S and observer O for $v \rightarrow c$. For $v \ll c$ expressions (1) and (2) almost coincide, and thus by measuring a frequency shift the observer would not easily discriminate his motion from the motion of the light source. However, when $v \rightarrow c$, expressions (1) and (2) differ due to higher-order terms of the binomial expansions.

We now ask whether it is possible to obtain directly from Eqs. (1) and (2), as separately formulated, a combined expression that includes both equations such that "frame-of-reference" uncertainty (i.e., the inability to discriminate f' from f'') increases so as to be consistent with the principle of relativity as applied to lightwave propagation.

Frame-of-reference (macroscopic) uncertainty (i.e., the principle of relativity) implies that it is not possible for an observer (O) to predict exactly whether a source (S) or an observer (O) is moving at a given instant of time, for a given movement of S relative to O. It is only possible to predict that either of these two possibilities or a combination of these two will occur with a certain probability. Thus, frame-of-reference uncertainty implies that probabilistic term(s) must be included in order that a description of light-wave propagation will encompass the principle of relativity.

For an amplitude function in general the square of that function is most commonly considered to be proportional to the probability distribution of possible values of that function. We will seek, therefore, a mathematical step whereby a squared term (proportional to probability) for frequency, f^2 , can be obtained from the classical Doppler equations (1) and (2) just described.

Multiplying Eqs. (1) and (2), we obtain directly an expression that now includes the squared term f^2 :

$$f'f'' = \frac{1}{1 - \frac{v}{c}} f\left(1 + \frac{v}{c}\right) f = \frac{c + v}{c - v} f^2$$

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$$\sqrt{f'f''} = \left(\frac{c+v}{c-v}\right)^{1/2} f.$$
⁽³⁾

The principle of relativity specifies sufficient frame- of-reference uncertainty when f' cannot be discriminated from f'. Thus, to satisfy the principle of relativity we suppose that in expression (3), as $v \rightarrow c$,

$$f' = f'' = \sqrt{f'f''} = \sqrt{f'f'} = \left(\frac{c+v}{c-v}\right)^{1/2} f$$

or

$$f'_{S \to O} = \left(\frac{c+v}{c-v}\right)^{1/2} f.$$
(4)

Equation (4) describes the relativistic Doppler effect for light waves since it can readily be shown that this equation is equivalent to the more familiar formulation

$$f' = \frac{f}{1 - \frac{v}{c}} \sqrt{\left(1 - \frac{v^2}{c^2}\right)},$$

in which the classical formulation $f/(1-\frac{v}{c})$ is modified by the term $(1-\frac{v^2}{c^2})^{1/2}$ taken directly from the Lorentz transformations. However, in the derivation just described for the relativistic Doppler effect, no introduction of the Lorentz-transformation term γ was necessary. It was only necessary to apply the principle of relativity directly to the product of the equations for the classical Doppler shift in a functional expression for f^2 .

A similar non-Lorentzian derivation for the relativistic Doppler shift when S moves away from O or O moves away from S will produce the equation

$$f_{\leftarrow SO}' = \left(\frac{c-v}{c+v}\right)^{1/2} f.$$

which is also easily recognized as the more familiar formulation

$$f' = \frac{f}{1 + \frac{v}{c}} \sqrt{\left(1 - \frac{v^2}{c^2}\right)}.$$

In the derivation just described for the relativistic Doppler effect(s), the term γ emerged from the principle of relativity, as applied to the classical Doppler-shift equations to include macroscopic uncertainty with a functional expression for f^2 , when the condition $v \rightarrow c$ constrains that functional expression.

Macroscopic uncertainty for moving frames of reference is then included in the expression for f^2 with equivalence of movements of source toward observer and observer toward source. Relative-velocity terms present in the classical Doppler equations were included in the above derivation without consideration of their measurability by the observer.

It is known that special-theory effects become appreciable only when particles moving at high velocity begin to undergo physical changes that include wave-particle duality and microscopic uncertainty described by quantum theory. To further assess the contribution of this constraint to the relativistic description, we now apply quantum theory to the classical Doppler formulation.

Case 2. In a second derivation (without the Lorentz transformation) of the relativistic Doppler effect, we will attempt to include some consideration of probabilistic aspects of photon distribution. For this second derivation, we again begin with a classical formulation, the simple sinusoidal wave function as applied to reflection of light waves by a mirror moving with velocity v with respect to the light source. For the electric field E of a plane light wave incident on a mirror moving exactly normal to the direction of light propagation, the following expressions are valid:

$$E_{\rm i} = a_{\rm i} \cos\left(\omega_{\rm i} t - k_{\rm i} x\right)$$

for the incident wave,

$$\vec{E_r} = \vec{a_r} \cos(\omega_r t + k_r x)$$

for a totally reflected wave.

For $v \ll c$, x = -vt, $E_i = E'_r$ at the mirror we obtain

$$a_{i} = -a'_{r}, \quad \frac{\omega_{i}}{k_{i}} = \frac{\omega_{r}}{k'_{r}} = c,$$

which will result in (see [2])

$$\omega_{\rm r} \left(c - v \right) = \omega_{\rm i} \left(c + v \right)$$

or

$$\frac{\omega_{i}}{\omega_{r}} = \frac{c - v}{c + v} = \frac{2\pi f_{i}}{2\pi f_{r}} = \frac{f_{i}}{f_{r}}$$

or

$$f'_{\rm r} = \frac{c+v}{c-v} f_{\rm i} \,. \tag{5}$$

This is a formulation for the classical Doppler shift for a mirror moving toward and normal to the direction of the propagated light wave.

It should be noted that only the Doppler shift for the reflected light wave itself, i.e., considering the moving mirror as the source, is described by (5). For the Doppler shift considering a "virtual source" behind the mirror, the velocity of the mirror would equal 2v. To make *Case 1* and *Case 2* more directly comparable, however, the Doppler shift is considered here for the reflected wave as if it were emitted from a source that moves in the same direction as the incident wave that is emitted by the stationary source.

To broaden the generality of this classical Doppler effect we now consider conditions for which $v \rightarrow c$, rather than conditions limited only to $v \ll c$, as specified so far. Actual physical conditions for which $v \rightarrow c$ require that we treat atomic and subatomic particles that can move at high velocities, i.e., $v \rightarrow c$, instead of a "moving mirror." For these purposes, an atom (or an electron) that radiates light can be considered as a light source that is equivalent to the moving mirror for which the classical Doppler effect was just described.

Atomic (and thus electron) emission and absorption of light occur in discrete quanta as first described by Planck [3]. The Rayleigh-Jeans law accounts for the energy distribution actually observed for black-body radiation when $hv \ll kT$. Thus, for the amount of energy U(v)dv we have the expression

$$U(\mathbf{v}) d\mathbf{v} \approx kT \mathbf{v}^2 d\mathbf{v}$$

where k is the Boltzmann constant, T is the temperature, h is Planck's constant.

For higher radiation frequencies, however, Planck's distribution is necessary [3], namely,

$$U(v) = \frac{8\pi v}{c^3} h v^3 \frac{\exp\left(-\frac{hv}{kT}\right)}{1 - \exp\left(-\frac{hv}{kT}\right)}$$

The Planck's-distribution equation is based on the requirement that light energy is absorbed or emitted (as for our particle source) in quanta and the energy E is equal to nhv, where v is the light frequency.

Light quantization within this formulation, i.e., Planck's law, is essential to describe the absorption or emission of light energy of higher frequencies by harmonic oscillators such as electrons within the walls of a hollow cavity. However, from Einstein's analysis of the photoelectric effect (see **Discussion** below), the frequency of light (but not its intensity) that is absorbed by electrons within a metallic surface is related directly to the velocity of the electrons induced to "escape" by the light, namely,

$$h(v-v_0) = \frac{1}{2}mv^2$$
,

where v_0 is the minimum frequency for electron escape.

Thus, for higher radiation frequencies (such as those accounted for by Planck's black-body energy distribution), we can infer that much higher electron velocities will also be encountered in the photoelectric effect (or the inverse photoelectric effect – see **Discussion**). This will be especially true for frequencies > 10^{18} Hz, as occur for gamma rays. Therefore, for $hv \gg kT$ (black-body radiation) $v \rightarrow c$ (the photoelectric effect).

Furthermore, a similar relation must also be obtained for the velocity of radiating electrons within orbital trajectories around atomic nuclei. Namely, higher radiation frequencies (i.e., $hv \gg kT$) will resonate with higher electron orbital frequencies and thus higher orbital velocities. Notwithstanding a "cloud"-like distribution of electron loci, for higher quantum numbers the relation $v = 2\pi\omega R$ approximates the correspondence of orbital electron velocity to frequency of oscillation.

To broaden the classical Doppler effect derived above for a moving mirror, therefore, we will seek to introduce Planck's law for radiation from an equivalent electron (or other particle) light source that can move at high velocities, i.e., $v \rightarrow c$. Introduction of light quanta into the classical Doppler formulation, then:

1) allows inclusion of higher frequencies and, thus, particle velocities;

2) includes probabilistic considerations (and thus microscopic uncertainty), since radiation of light quanta depends on the probabilities of transitions between atomic states;

3) by Bohr's principle of complementarity [4, 5] leads to a light intensity function (see below) that can be considered as an approximate measure of the probability of finding photons within an area of the wave flux.

Light intensity I, the average rate of energy transfer through unit area, can be considered proportional to E^2 and the average number of photons flowing through that area.

The intensity I_i of the light incident on the moving mirror can be described in terms of light quanta by

$$I_{i} = N_{i} h f_{i} , \qquad (6)$$

where N_i is the average total number of photons through unit area in unit time.

Similarly, the intensity of reflected light I_r is equal to

$$I_{\rm r} = N_{\rm r} h f_{\rm r} \,. \tag{7}$$

Returning to the formulation (see above) for the classical Doppler effect for the moving mirror, we get

$$\frac{f_{\rm r}}{f_{\rm i}} = \frac{c+v}{c-v} \,.$$

Since N_i must equal N_r for totally reflected light, we obtain from Eqs. (6) and (7)

$$\frac{I_{\rm r}}{I_{\rm i}} = \frac{N_{\rm r} h f_{\rm r}}{N_{\rm i} h f_{\rm i}} = \frac{c+v}{c-v} \,. \tag{8}$$

A similar relation was originally obtained by Wien using thermodynamic arguments to derive his "displacement" law (cf. [6]), which accounts for high-frequency black-body radiation.

According to quantum theory, the emitted light quanta (as just described) are part of a system that includes the emitting atomic or subatomic source. Bohr's correspondence principle allows another description of energy emission, one that is derived from classical descriptions of harmonic oscillators. Harmonic oscillators can be particles (electrons) or quantized packets within an electromagnetic field. In the correspondence limit, the average energy transfer will correspond to energy resulting from summation of harmonic oscillators that in the classical limit are approximated in a simple form by descriptions for waves in general. Waves in general can be considered as oscillating systems described by an expression for the energy of oscillation, namely, $\frac{1}{2}\omega^2 A_0^2$ or $\frac{1}{2}(2\pi f)^2 A_0$, where A_0 is the amplitude (peak); f is the frequency of oscillation.

It can also be shown, however, that the energy W of the radiation itself, i.e., the electromagnetic field, can be represented as the energy of a group of harmonic oscillators:

$$W=2 \sum f^2 q q^*,$$

where q^* are dynamical coordinates (that specify the radiation field).

This latter formulation is obtainable from classical electromagnetic theory. However, these same terms satisfy a wave equation and thus can be encompassed by quantum theory. To further describe harmonic oscillators with reference to quantum theory, classical approximations (for Schrödinger wave equations [7]) can be used in perturbation theory. Thus, vector potential(s) from Maxwell's equations can be related to the light intensity I. Beginning with Poynting's vector, an expression for the intensity of the radiation I can be obtained:

$$I = \left| \frac{c \; (\mathbf{E} \times \mathbf{H})}{4\pi} \right| \, ,$$

where

$$\mathbf{E} = -\frac{1}{c} \frac{d\mathbf{a}}{dt}; \quad \mathbf{H} = \nabla \times \mathbf{a};$$

a is the vector potential, specified precisely for each point in space and time.

Recalling that in free space

$$|\mathbf{E}| = |\mathbf{H}|, |\mathbf{E}| \times |\mathbf{H}| = |\mathbf{E}|^2,$$

it can be shown that the time-average intensity is equal to [8]

$$I = \frac{\omega^2}{2\pi c} \left| \mathbf{a}_{\mathrm{o}} \right|^2.$$

We now have, therefore, two expressions for the intensity of the radiation I:

$$I = Nhf, \quad I = \frac{\omega^2}{2\pi c} |\mathbf{a}_0|^2.$$

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According to the complementarity and correspondence principles, these two expressions must apply to the energy not only within the electromagnetic (quantized) waves but also within the source of these waves such as atomic or subatomic particles capable of moving at high velocity. Such a particle source can be considered as approximately equivalent to a moving mirror that totally reflects incident light. However, the particle source in the correspondence limit can move at velocities not encompassed by the nonrelativistic Doppler derivation shown above for the moving mirror. Thus, since classical approximations of Schrödinger wave equations used in perturbation theory only apply to particle or radiation oscillators, the condition $v \rightarrow c$ can be included in these approximations.

Returning, therefore, to consideration of the moving mirror, now treated as an equivalent particle light source, for incident and reflected light we write

$$I_{\rm i} = \frac{\omega_{\rm i}^2}{2\pi c} |{\bf a}_{\rm io}|^2, \ I_{\rm r} = \frac{\omega_{\rm r}^2}{2\pi c} |{\bf a}_{\rm ro}|^2$$

Dividing I_r by I_i , we obtain

$$\frac{I_{\rm r}}{I_{\rm i}} = \frac{\omega_{\rm r}^2 / 2\pi c}{\omega_{\rm i}^2 / 2\pi c} \frac{|{\bf a}_{\rm ro}|^2}{|{\bf a}_{\rm io}|^2} = \frac{f_{\rm r}^2}{f_{\rm i}^2} \frac{|{\bf a}_{\rm ro}|^2}{|{\bf a}_{\rm io}|^2}.$$

Considering $a_{ro} \cong a_{io}$ and the above relations, in the correspondence limit, assuming an isotropic light source and an intense light beam, we can write

$$\frac{I_{\rm r}}{I_{\rm i}} = \frac{c+v}{c-v} \approx \frac{f_{\rm r}^2}{f_{\rm i}^2} \tag{9}$$

and thus,

$$f_{\rm r} \approx \left(\frac{c+v}{c-v}\right)^{1/2} f_{\rm i} \,. \tag{10}$$

This relativistic Doppler equation emerges from the correspondence of light energy to the energy of equivalent radiation oscillators that are treated as equivalent to an emitting electron source that can move at high velocity (i.e., $v \rightarrow c$). Classical approximations of Schrödinger wave equations (from perturbation theory) were required to implement this correspondence. These classical approximations are completely rigorous only for intense light beams that represent macroscopic collections of large numbers of photons [8]. We may consider that the initial inclusion of quanta and the accompanying microscopic uncertainty implicit in these wave-equation approximations that have macroscopic meaning transform the classical Doppler equation into the relativistic result.

Discussion. The equivalence of quantal and corresponding classical descriptions of energy was originally suggested within Einstein's analysis of the photoelectric effect.

Thus, we can infer three sequential steps in the photoelectric effect:

1. Light energy in the oscillations of the quantized electromagnetic field is transferred to the electron oscillators within the metallic surface.

2. Energy of the electron oscillations reaches a threshold for the electron's escape from its atomic orbit.

3. Upon escape, oscillation energy is transformed into kinetic energy of the escaping electron(s) moving at high velocity.

In summary, we have

$$h(f-f_0) \rightarrow \Delta\left(\frac{\omega^2}{2}mx_0^2\right) \rightarrow \frac{1}{2}mv^2$$
.

The above yields the relativistic Doppler formulation emerging in Case 2 just described. It is critical that the derivation of Case 2 included higher electron velocities (i.e., $v \rightarrow c$) by including the higher radiation frequencies within Planck's energy distribution. Quantal descriptions of radiation (hv) must encompass relativistic velocities of radiation-source electrons as they orbit atomic nuclei. The quantal radiation energies, therefore, are an integration of classical as well as relativistic orbit velocities for electrons that absorb or emit that radiation.

For both the photoelectric effect and its inverse, however, there is a crucial interaction of light and matter (the electron). This interaction permits exchange of energy when the quantized light oscillators resonate with the electron oscillator(s) within atomic orbits. The derivation of *Case 2* (see above) suggests that relativistic Doppler effects depend on this light–electron transfer of energy within resonant wave oscillations. In this transfer, the overall intensity of the electric energy (and thus the number of electrons), but not the velocity of individual electrons, varies with the intensity of the light (and thus the number of quanta). Similarly, the resonant frequency of individual electron oscillators depends on the frequency of the light but not the intensity.

We might ask how is it possible to have two intensity functions such that in one (8) the intensity is proportional to the frequency, and in the second (9) the intensity is proportional to the square of the frequency. Bohr's principle of complementarity in fact provides that the intensity of electromagnetic waves can be interpreted in two ways. One way describes the intensity as the number of quanta or photons passing through unit area per unit time, and for this way the intensity varies as the frequency. The second interpretation of the intensity (according to Bohr) describes the intensity for waves as proportional to the square of the electric-field intensity E^2 (and H^2). For this way, the intensity varies as the square of the amplitude or, for a constant amplitude, as the square of the frequency. Thus, Bohr's complementarity principle implies two ways of describing the intensity and, thus, two expressions, one proportional to the frequency, the other to the square of the frequency. The latter expression becomes more explicit and relevant when perturbation theory is applied to quantized radiation oscillators that are in resonance with the electron oscillators (within atomic particles). Perturbation theory, as applied here, however, utilizes classical approximations of Schrödinger wave equations (see above), which account for "particles" of matter as well as light. In general, the Schrödinger equation [7] is a wave function formulated for $\Psi(x, t)$ such that $|\Psi(x, t)|^2$ is the probability of finding the particle within the distance interval dx at x. This is precisely how the square of the electric-field intensity E^2 multiplied by the volumetric interval dV is proportional to the probability of finding a photon (or a quantum of light) within that interval.

The relativistic Doppler shift as derived above describes the shift of the light intensity (or the energy transmission through unit area) due to the velocity of the light source. This source-velocity-induced intensity shift depends on the quantal description of electromagnetic-wave radiation even at the highest frequencies and on Bohr's complementarity and correspondence principles. As discussed above, these principles encompass microscopic uncertainty for the relativistic Doppler formulation when the probability of encountering a photon is approximated by the number of quanta of a given frequency N in a specified distance interval and the square of the electric-field intensity E^2 within a specified volume interval.

The probability of encountering photons of relativistic-Doppler-shifted frequency, therefore, emerges from application of quantum theoretic principles with their included microscopic uncertainty. Under certain conditions, e.g., high quantum number (n) and/or high number (N) of quanta, quantal descriptions correspond to a macroscopic electromagnetic flux. By contrast, macroscopic uncertainty due to moving frames of reference (*Case 1*) yields a relativistic Doppler formulation when the principle of relativity is applied to light waves.

These Doppler derivations were implemented in the absence of the Lorentz transformations. Furthermore, other equations essential to the special theory of relativity can be derived directly from the relativistic Doppler equations as derived here in the absence of the Lorentz transformations. One interpretation of these derivations suggests, therefore, that relativistic considerations themselves derive directly from Bohr's principles of correspondence and complementarity. When classical descriptions "correspond" to and include quantal descriptions, according to this interpretation, relativistic modification of the relevant equations will have also been included.

The necessary role of microscopic uncertainty in relativistic phenomena, as implied above, emphasizes the fundamental role of physical transformations in these relativistic effects rather than measured differences due to different frames of reference. Radiation of an emitting electron, for example, will have a wavelength that decreases (according to the De Broglie relation [9]) and a mass that increases as the electron's velocity approaches c. Increased mass and frequency with increased particle velocity are closely related aspects of matter's conversion into energy that can only be described probabilistically.

In summary, as a radiating particle accelerates to velocities that approach the speed of light, it begins to oscillate with increasingly higher frequency. This progressively higher frequency and reduced period of oscillation introduce progressively more uncertainty into its path of motion. Concomitantly, the light emitted by the accelerating particle is of progressively higher frequency (according to the Doppler effect), a frequency that must have some resonance with the frequency of the electron's oscillation(s). A progressively greater percentage of the energy transferred to the particle through its acceleration is in the form of increased frequency of oscillation (or "mass"). Thus, the high velocity particle becomes more "wave-like" in its changes in energy as its velocity approaches the speed of light and as it enters the domain of microscopic uncertainty and thus quantum theory. According to this framework, the relativistic Doppler effect derives from energy shifts (induced by high source velocities) that must be considered within the quantum theoretic domain.

Thus, these derivations of a relativistic Doppler shift suggest that the present theory more directly describes intrinsic probabilistic properties of electromagnetic radiation rather than measurement differences attributable to moving frames of reference.

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